- Dedicated to Professor Mario Castagnino on the occasion of his 75th birthday.

Gravity and Nonequilibrium Thermodynamics of Classical Matter

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(Dated: October 20, 2010)

Abstract

Renewed interest in deriving gravity (more precisely, the Einstein equations) from thermodynamics considerations [1, 2] is stirred up by a recent proposal that 'gravity is an entropic force' [3] (see also [4]). Even though I find the arguments justifying such a claim in this latest proposal rather ad hoc and simplistic compared to the original one I would unreservedly support the call to explore deeper the relation between gravity and thermodynamics, this having the same spirit as my long-held view that *general relativity is the hydrodynamic limit [5, 6] of some underlying theories for the microscopic structure of spacetime* – all these proposals, together with that of [7, 8], attest to the emergent nature of gravity [9]. In this first paper of two we set the modest goal of studying the nonequilibrium thermodynamics of classical matter only, bringing afore some interesting prior results, without invoking any quantum considerations such as Bekenstein-Hawking entropy, holography or Unruh effect. This is for the sake of understanding the nonequilibrium nature of classical gravity which is at the root of many salient features of black hole physics. One important property of gravitational systems, from self-gravitating gas to black holes, is their *negative heat capacity*, which is the source of many out-of-the ordinary dynamical and thermodynamic features such as the non-existence in isolated systems of thermodynamically stable configurations, which actually provides the condition for gravitational stability. A related property is that, being systems with *long range interaction*, they are nonextensive and relax extremely slowly towards equilibrium. Here we explore how much of the known features of black hole thermodynamics can be derived from this classical nonequilibrium perspective. A sequel paper will address gravity and nonequilibrium thermodynamics of quantum fields [10].

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Happy Birthday Mario, the philosopher, the king and the philosopher king! – This was explained in the first few slides of my talk at Mariofest. In an earlier, different occasion I had compared Mario, in his capacity of an inspiring mentor and a chief architect, in building up an eminent school of theoretical physics and astrophysics in Argentina, to my own Ph. D. advisor, the late Professor John Archibald Wheeler, in the U.S.A. The nature of this meeting could perhaps allow me to also relate some of my past experience with Wheeler, and to pay homage to his influence on me in the same capacity as is done here by many young researchers, leaders in their own rights in different fields of physics and astrophysics, with Mario. So please forgive me if you find me delving at times into the past, referring to what I was thinking when I was a graduate student, some 40 years ago, on certain topics, some still of current interest. One of these ideas bears on the present theme of gravity in relation to thermodynamics, another on the philosophy I use for understanding it.

I. INTRODUCTION

After Deconstructing Quantum Gravity, - What is Gravity? What is the Quantum?

This is the title of my talk but this essay addresses only the first question therein, namely, what is gravity, viewed in the light of gravity as an emergent phenomenon, and taking gravity as the thermodynamic limit of some microstructure of spacetime. I endorse this viewpoint as it is on the same footing as my long-held view that general relativity (GR) is the hydrodynamic limit [5] of some underlying *theories for the microscopic structures of spacetime*, the italicized words are what I advocate as the new meaning of quantum gravity (QG). This thermodynamic view of gravity which Jacobson [1, 2] started has caught the attention of a broad audience after Verlinde's recent proposal that "gravity is an entropic force" [3] (see also [4]). Although I find his reasonings in the derivation of Newton's second law and the law of universal attraction (drawing on quantum physics in the capacity of Bekenstein entropy [11] and Hawking radiance [12] in black holes, quantum information tied to the holography principle [13] and the Unruh effect [14] used to relate acceleration to temperature) somewhat overladen and simplistic I do support furthering our understanding of the relation between gravity and thermodynamics, examining how Einstein's equations arise in this emergent view and using the fact that GR is the theory of gravity for our

macroscopic world to derive possible thermodynamic principles such as maximum entropy which can be applied to obtain the thermodynamics limit of quantum gravity. In this first paper of two we set the modest goal of studying the nonequilibrium thermodynamics of classical matter only, bringing afore some interesting yet perhaps not so well-known prior results from classical physics to illustrate the nature of gravity, how many salient features of it are common to other interactions and how gravity is different.

To appreciate this new paradigm towards gravity I need to first summarize the key ideas in my view of quantum gravity (or how it is deconstructed) so as to shape the new perspective for understanding classical gravity (this follows immediately below), and describe what the goal of this paper is, namely, to expound gravity from a nonequilibrium thermodynamics viewpoint, treating it as a classical system with negative heat capacity (NHC) or as belonging to systems with long range interactions (LRI) and finding out what in gravity is in common with other systems with these properties. We do this in Sec. 3. In Sec. 4 we will discuss black hole thermodynamics from this angle, asking if one could derive the Bekenstein entropy formula from a collapsing shell of self gravitating gas, whether the nonextensive character of LRI systems (LRIS) can yield an area- rather than the usual volume- dependence (extensivity) of entropy, and what the failure of the zeroth law of thermodynamics in such systems implies with regard to the second law of black hole thermodynamics. There is a reason why I want to do it in this methodical way from first principles, namely, to fix the one primary progenitor idea (statistical mechanics of LRI systems) and place the others as secondary or tertiary derived ones. This minimalist attitude I adopt is based on two principles: the Austerity Principle of Wheeler [15] and my Commonality Principle explained in Sec. 2. A critique of this recent proposal of Verlinde can be found in the Appendix. In a sequel paper [10] I will discuss gravity with quantum fields from a statistical mechanics perspective and what the gauge/gravity duality implies in this new viewpoint.

A.   What is Quantum Gravity?

Despite the multiplying brands and divergent approaches, from quantum general relativity to loops to string theories and causets [16], I hope an agreement can be reached soon on a new definition of *quantum gravity (QG) as "a theory for the microscopic structure of spacetime and matter"* and with it I would propose stop using quantum gravity in the nar-

row (and likely misleading) sense of "quantizing gravity". The reason is because the only theory of gravity we know and trust is the theory of general relativity (GR) but quantizing GR does not necessarily lead to QG in the sense defined above. Yet, for 60 yrs this has been the prevailing opinion and practice in the GR community: that a theory for the microscopic structure of spacetime and matter (QG) can be obtained by quantizing general relativity. Most research activities were directed in those decades towards seeking better quantization variables and nicer mathematical formulations. I have had doubts on these programs from the beginning because they all made a *tacit yet unproven assumption*, that quantization of the metric or the connection forms which are macroscopic variables in Einstein's GR theory, leads to a theory for the microscopic structures of spacetime. This crucial assumption was never justified, and from what I know in other areas of physics, is generally not true. A different way of thinking with a totally new paradigm is called for. I view general relativity as valid only in the hydrodynamic limit of some (as yet unknown) microscopic theories and as such it is not a fundamental theory, but an emergent one. The string theory community's view is clear from the beginning: the microscopic theory is string theory. In recent years they announced that gravity is emergent [17, 18], but how spacetime emerges from string interactions has yet to be shown. Similar claims are made for loop theory [19, 20], and therefore, they face the same challenge of showing e.g., the existence of Minkowski spacetime as a stable vacuum at low energies. For the latest progress in causal sets approach see, e.g., [21].

Since the main ideas of my view on this subject can be found in earlier papers [5, 6, 22–25] I will just list some key points and mention their implications as follows.

a. 1. GR as Hydro, metric and connection forms are collective variables Direct Implication: *Makes no sense to quantize GR.* Doing so will only get the quantized collective degrees of freedom like in phonon physics, not atomic structure.

Note that hydrodynamics here refers to the long wavelength low energy limit of a microtheory describing the substructure of spacetime and matter, not the hydrodynamics of conformal fields in the context of the fluid/gauge duality.

b. 2. Gravity and gauge fields are macro objects, both are emergent from the same micro-theory at its hydrodynamic limit(s). The existence of a duality or correspondence between them shows that they stand on the same footing.

Some leading string theorists point at AdS / CFT correspondence as an indication that

gravity is emergent. To me this is not hitting the nailhead. AdS is not emergent from gauge fields nor the converse. **Correspondence is not emergence.** Gravity and gauge theory could be different manifestations of the same underlying, more fundamental microscopic theory in different parameter regimes.

c. 3. Macro-structure is largely insensitive to the details of the underlying micro-structures. Many micro-theories can share the same macro-structure. It is the collective properties of these micro structures which show up at the long wavelength, low energy limit. In fact one should at the lowest order approximation *look first at the commonalities of all competing micro-theories rather than their differences*, namely, their hydrodynamic limits rather than the detailed micro- (string or loop) behavior, in the same sense that magneto-hydrodynamics is based on atoms or ions and magneto-chromo-hydrodynamics based on quarks and gluons share similar features in this limit, namely, hydrodynamics. The micro-theories could belong to different universality classes similar to the situations in critical phenomena. A classification of these micro-theories in terms of their critical behavior would be very useful in identifying our particular universe. One could then ponder on what other universes could form from different micro-theories under different conditions in their hydro-dynamic limits.

d. 4. New tasks of quantum versus emergent gravity defined: The task of quantum gravity is to induce / discover the micro-structures of spacetime from the known macro-features of our universe while the task of emergent gravity is to explain how these known macro-features of our universe emerge from the assumed micro-theories. I describe this in a recent essay [6]. For the conceptual development of emergent gravity, see [9].

II. COMMONALITY PRINCIPLE

As remarked in the beginning though sharing the same view that gravity is emergent and GR is the thermodynamic and hydrodynamic limits of some theories for the microscopic structure of spacetime (quantum gravity) I find the arguments backing up the proposal of gravity as entropic force [3] overly simplistic and contrived. As the proposer acknowledged, all the ingredients used are known before, what is different here is just placing a different emphasis on which principles appear to be more fundamental in nature. Since there is no new results reported compared to the original proposal [2] it boils down to a comparison

of the practicing principles and the intellectual attitudes taken in the different approaches. The criterion for deciding what concepts are regarded as primary and fundamental versus what others are secondary and derived is a philosophical issue. Thus foremost we need to recognize differences at the philosophical level – after all, this is how a viewpoint is formed.

We prefer to evoke the minimal amount of thematic material and work with the lowest common denominator which gravity shares with other physical systems or forces in nature. This philosophy encapsulated in the so-called "Austerity Principle" or "Commonality Principle" explained below is in stark contrast to the "mix and fix" or "chop-sui" approach [64] to formulating new ideas which seems to be in vogue these days.

'Common' here connotes at least the following three senses: a) that it is not so unusual, particular or outstanding: It conveys the intent of *seeking a more basic level of understanding*, and in so doing recognize that some new alleged discoveries can in essence be more commonplace than when they are first conjured. b) Common in the sense of common denominator: *seeking a common ground with other apparently unrelated phenomena*, so as to be able to see their universality. 3) Common as in commoners: Principles for Commoners. The philosophy behind Wheeler's *"poor man's way" of looking at the essence of matter or getting to the heart of a problem* smacks of this: Find a way that even a poor man can come to appreciate what is valuable in what we want to say.

In applying the Commonality Principle we can usually deconstruct what seems too complex or demystify what appears so intriguing. Only by performing this screening using these criteria, by first finding out what characteristic features are in common with other physical phenomena, can we identify what is truly unique and special about a new discovery, in the present case, a new way of understanding gravity in contra- distinction to other forces. The price one pays for maintaining a self-presumed special stance or unique status is, as we know from the difference between aristocrats and commoners, that despite their imposing appearances or staged elegance they often don't get much done in what really matters, because they are restricted by their own perceived superiority and artificial distinction.

There are many examples we can think of in the history of development of ideas in physics, some more recent examples are: Calabi-Yau space as containing the fermion spectrum in the unification scheme between spacetime and particles. At its inception people thought there was only one such space which matches exactly what we observe in Nature, but of course it was later found that there are millions. Likewise for the quasi-classical domain after

decoherence of a quantum system. In the beginning people thought that it is enough to just perform one set of projections in the decoherence functional and then the physical world appears. But later it was found to have infinitely many. String theory is a deep theory but so far rather remote from observation, but the AdS/CFT correspondence [26] which was inspired by it took on a thriving life of its own. Yet physics is not quite represented by supersymmetric Yang-Mills theory at the boundary of our universe, so I would take it as *suggestive* of an interesting and important relation rather than taking all the results of AdS and CFT calculations *literally.* I believe the gauge/gravity duality [17] which is a representation of this correspondence will produce more useful results because it relates the two well-established and well-understood theories which form the cornerstones of theoretical physics, in today's familiar low energy rather than in the lofty Planck energy domain.

As a familiar example in gravitation theory, recall Unruh's way of understanding Hawking radiation from accelerated detectors (1976) [14] or from his fluid model (1981) [27]. The essential physics in black holes or what special effects may originate from the event horizons can be understood through more commonplace and reachable physical phenomena. I share the same philosophical perspective. At the time, the prevailing view of the GR community is that Hawking and Unruh effects are due to the existence of an event horizon which is a global property of spacetime. This is absolutely correct. But if one deviates slightly from that setup and asks the following question: *Is there radiation when the detector is accelerating but not uniformly?* 1) Mathematically, there is no event horizon for such state of motion and the answer should be NO, if we place the presence of an event horizon as a criterion for the existence of radiation. However, 2) physically, this is a common practice (of say, passing a slow driver, going from uniform velocity to uniform acceleration then back to uniform velocity) and a continuous extension of the uniform acceleration case. So one would expect that there is radiation, but not in a thermal form. The traditional way of using global concepts is too rigid for this problem. A broader way of thinking with a new method is better suited. We did a calculation using stochastic field theory [28] for finite-time uniform accelerated detector interacting with a quantum field and found that 2) is the correct answer. This method enables us to calculate the spectrum of radiation from any arbitrary trajectory, no temperature concept is needed because the system is under nonequilibrium condition.

III. GRAVITY AS NONEQUILIBRIUM THERMODYNAMICS OF CLASSICAL MATTER

A. An aside: Some old yet still stimulating ideas

In the year 1969-70 when I began my doctoral work with Wheeler four ideas crossed my path and stayed on my mind ever since: 1) Sakharov's one page 1967 paper about metric elasticity [29] which later led to induced gravity and inspired my view that GR is a hydrodynamic theory. 2) Wheeler's riddle-like "boundary of a boundary is zero" motto, which is explained in Chapter 15 of MTW [30] and years later further developed into his Austerity Principle [15] 3) Lynden-Bell's paper with Wood [31] on the negative heat capacity of gravitating systems which I will expound further with you in this paper and 4) spectral decomposition of the Laplacian. Wheeler introduced the work of the mathematician M. Berger to me, wanting me to use it to see if there is some special state in the mixmaster universe similar to the magic number in the nuclear collective model of Hill and Wheeler, Bohr and Mottleson. All four ideas share one common feature on this curious and uninitiated young mind: unconventional and oblique, yet elegant and appealing; deceptively simple in appearance but probably loaded with deeper meanings. Strange things usually stay longer in one's memory.

Here's a little background: For 1) at that time the big thing in GR was the singularity theorem of Penrose, Hawking and Geroch. Every serious student in general relativity needs to be conversant in this vigorous mathematical enterprise called global analysis. So where should one place this idea of Sakharov, fallen from the blue, totally out of line? For 2) what Wheeler wanted to convey is that almost all important equations in theoretical physics from QED to QCD to perhaps QGD (quantum gravi-dynamics), those which embody the Bianchi identity, seem to convey something rather obvious, if not 'vacuous', because that identity basically states that the boundary of a boundary is zero. This identity signifies the conservation of the 'moments of rotation' of a geometric object which corresponds to the conservation of energy-momentum tensor of matter by way of the Einstein equations. Wheeler later developed this idea further and summarized his understanding in the so-called 'Austerity Principle' (essentially saying, if I may use a Wheeleresq expression: that 'nothing' is 'everything') This conservation law is what inspired me later to finally affirm that general

relativity is geometro-hydrodynamics [5] (there were earlier hints I noted from cosmology as 'condensed matter' and semiclassical gravity as mesophysics [23]), because, just like hydrodynamics is the long wavelength, low energy limit of the underlying microscopic theories of matter, a geometric (manifold) description of spacetime structure with its dynamics described by the theory of general relativity is meaningful only in the hydrodynamic regime of a microscopic theory of spacetime, which is the proper definition of quantum gravity. This is further corroborated by the findings at that time by Hartle, Laflamme and Marolf [32] that a quasi-classical domain of the hydrodynamic variables can exist after decoherence projections because there exist conservation laws that these variables obey.

For 3) we learned that heat capacity for ordinary matter is always positive, it is the condition for the stability of the canonical ensemble. (We have to know this to pass the gruesome yet empowering General Exam.) Now this English gentleman astronomer is telling us despite everything is coupled through gravity, gravitational force behaves opposite to almost everything. I wanted to find out more about this idea, it appears to be the overriding property of all gravitational phenomena, from self-gravitating gas to black holes. I didn't get to it though, because I need to study quantum field theory in curved spacetime! So I had to put this quest on hold, pretty much until now.

4) is engrained in "Can you hear the shape of a drum?" I started reading the series of papers by Balian and Bloch, where they tried to apply this formula to nuclear properties. What stuck on my mind was that this is one neat way to connect the big with the small: the geometry and topology of space from the eigenvalues of the invariant operators defined on it. The more I studied this the more I saw its beauty, but got totally frustrated because this formula applies only to Riemann and not to pseudo-Riemann spaces, but I need to construct a formula for the latter to be able to say something about cosmology. If I were a bit less ambitious and complacent with spacetimes with Euclidean sections I could have gone down the route of discovering the zeta function regularization method and identified the 'corner' term in that expansion as the a_2 HamideW coefficient which enters in the conformal anomaly! (I learned later that great advisors always push their students beyond the limit, not only of their own ability, but also beyond what is commonly perceived as acceptable at the frontier of research). This last point is very powerful – I saw a connection between geometry-topology and statistical mechanics, because it provides a microscopic description for the global properties of spacetime, in terms of the level density of an invariant operator.

I will return to this point in my sequel essay [10].

B. Old physics: Gravitational systems have negative heat capacity

Hereafter we will focus just on idea 3). To me the best introduction is still Lynden-Bell's [33]. The story began with Antonsov's 1962 'gravothermal catastrophe' [34]: When he analyzed the thermodynamics of a system of N particles of energy E in a spherical box of radius R he showed that there is no global maximum entropy state, meaning that the system is unstable thermodynamically. A local (meta-stable) maximum entropy state exists when R is not too large. When R is increased the density of the gas at the edge drops compared to that at the center. When this ratio decreases below 1/709 this meta-stable state disappears and the system becomes thermodynamically unstable. Lynden-Bell and Wood [31] offered the explanation that this is because gravitating systems possess negative heat capacity. Relative stability of stars is made possible by the balance of another force which provides positive heat capacity such as the main sequence stars fueled by thermonuclear reaction at the core. But the outer part of the star, like a self-gravitating gas, still has negative heat capacity. Energy released from the core is absorbed by its outer region expanding and cooling. Thus, according to Posch and Thirring [35], it is the thermal instability of gravitating systems which extinguishes the nuclear fire and keeps the star stable. Yet, as we all know, there is an end to nuclear reaction at which point gravity wins and after a supernova explosion the core turns into a neutron star or black hole, depending on the initial mass of the system. For higher mass systems beyond the Chandrasekhar limit the degenerate Fermi gas equation of state is too soft to resist gravity and this 'gravothermal catastrophe' leads to black holes.

All this about the thermodynamics of gravitating systems and gravitational collapse should be familiar to the gravity and astrophysics community. For a review, see e.g. [36]

The thermodynamical properties of gravitating systems are different from what we usually encounter for ordinary matter, where the use of canonical (CE) ensemble (systems in contact with a large heat reservoir) for their description is justified. For gravitational systems as well as some other kinds named below, one needs to revert to the use of microcanonical (MC) ensembles (for isolated systems at a definite energy E.) Thirring in 1970 [37] showed that two systems with negative heat capacity (NHC) cannot be in thermal equilibrium. Thus it

is impossible to find an equilibrium canonical ensemble for the combined system. This may look strange because our usual notion tells us that if one system A is in equilibrium with another B and if a third one C is found to be in thermal equilibrium with B, then C should also be in equilibrium with A. This is canonized as the Zeroth Law of Thermodynamics. This law implies the existence of an intensive quantity, the temperature. Notice that the validity of the Zeroth law has an implicit presupposition, that all systems concerned have positive heat capacity and have properties adapt to a canonical ensemble description. These more commonly encountered systems where our notions of thermodynamics are largely derived from are usually large systems with short range interactions. This is not the case for gravity. *Gravitational systems are nonextensive.* As discovered later, gravity is not alone in this regard: systems with long range interactions or small clusters – small compared to the interaction range – have similar thermodynamic properties.

Whether the Zeroth Law is obeyed by systems with NHC is an interesting issue. Even though we know these systems are thermodynamically unstable when they interact with their surroundings and anomalous behavior may ensue, it is not obvious that this violates the Zeroth Law, because while there is coupling, heat exchange is allowed and one cannot rule out the possibility that the total system may become canonical. This is what Ramirez-Hernandez et al [38] set forth to prove, using a small system of rotors in two dimensions treated both in the MC and in the canonical ensembles. They confirmed what Thirring proved earlier, that indeed the Zeroth Law conjured for canonical ensembles does not apply to systems with NHC. When two identical subsystems of NHC with the same intensive parameter (temperature defined in the MC sense) are thermally coupled they undergo a process in which the total entropy increases irreversibly. The intensive parameters of the two subsystems remain equal but that of the combined system is different from either subsystem. The two subsystems cannot maintain stable thermal equilibrium.

Knowing that black holes are systems with NHC this result of entropy increase when applied to black holes becomes a statement of the second law of BH thermodynamics. If we follow Bekenstein in assigning an entropy to the black hole as proportional to its area, then when two black holes are brought in contact with each other the combined system would see an entropy increase, which would signify an increase in the area of the merged black hole. The association of the area of a black hole with entropy stems from a different physics, perhaps best understood from the entanglement entropy viewpoint, to be discussed further

in my sequel essay. But as far as the entropy increase in the combined system is concerned, it can be understood as a consequence of both subsystems having negative heat capacity. We will continue to explore the thermodynamical properties of gravitating systems with and without an event horizon, to understand better the role played by the event horizon over and above their being systems of NHC.

C.   Features of gravity common to systems with long range interactions (LRIS)

Let's go a step further to seek the commonalities of black holes with other systems. Negative heat capacity is not unique to gravity. For systems with long-range interactions (LRIS), the two-body potential decays at large distances as $V(r) \sim 1/r^a$ with $a \leq d$, where d is the space dimension. Examples are: gravitational systems, two-dimensional hydrodynamics, two-dimensional elasticity, charged and dipolar systems. Their common properties are:

1. *Nonextensive / Non-additive*: In normal systems if one holds the intensive variables (temperature, pressure, chemical potential) fixed, then the extensive variables (energy, entropy) will increase in proportion to an increase in the size of the system. This is not true for LIR systems. Their extensive variables are intrinsically non-additive: sum of the energies or entropies of subsystems is not the same as the energy or entropy of the whole system.

2. *Inequivalence of Ensembles*, i.e., for the thermodynamics of such systems, results obtained from the microcanonical and canonical ensembles are different. This inequivalence implies that specific heat can be negative in the microcanonical ensemble, and temperature jumps can appear at microcanonical first order phase transitions. For a discussion of microcanonical thermodynamics, see, e.g., [39]. For different ensemble treatments of black hole thermodynamics, see the work of Braden, Brown and York [40].

3. For LRIS the space of accessible macroscopic thermodynamic parameters might be non-convex. The lack of convexity allows us to easily spot regions of parameter space where *ergodicity may be broken.*

4. LRIS also display an extremely slow relaxation towards thermodynamic equilibrium and *convergence towards quasi-stationary states.* This includes the famous class of glassy systems, which has been compared to the thermodynamics of black holes [41] (background temperature referred to there may be inappropriate [42]). See [43] for a review on the sta-

tistical mechanics and out-of equilibrium dynamics of systems with long-range interactions.

D.   New physics: Quantum Processes in Black holes and holography

Black hole thermodynamics is often viewed as holding the key to understanding the intersection, if not the union, of gravitation, quantum mechanics and thermodynamics. Bekenstein's identification of black hole surface area as entropy suggests a strong connection between gravitation and thermodynamics, while Hawking's discovery that black holes emit thermal radiation takes this to a higher level, incorporating quantum field effects in black hole thermodynamics.

Black holes also have negative heat capacity, no different from a gravitating gas. In the 80s there were discussions of this aspect in the works of Page, Penrose, Smolin, Sorkin and others. It is well known that a black hole in an asymptotically flat spacetime is unstable. To keep it in quasi-equilibrium we need to place a black hole in a box (of radius smaller than 3M) or in anti- de Sitter space where the curvature at infinity acts like a confining wall. Hawking and Page [44] showed that in an AdS space filled with thermal radiation the system can undergo a phase transition corresponding to the formation of a black hole. With the use of the AdS/CFT correspondence [26], Witten [45] showed that this transition would correspond to the deconfinement transition in QCD. These modern tenets of holography principle [13] and gauge/gravity duality [17] have catapulted black holes in AdS space into special prominence pertaining to both particle physics and gravitation theory. A lucid description of the thermodynamics of AdS black holes can be found in Hemming and Thorlacius [46].

Why do I pull out this 40 year-old topic of negative heat capacity for gravitating systems and place it next to the new fanciful developments of the last 10 years? Well, here is where the Austerity and Commonality Principles are at work. What I advocated in our thought process in relation to some recent hype is, figuratively speaking, to dig deeper at the roots, not just picking the fruits. The gravitational features of systems of interest involved in these new developments after 1970 – from black holes entropy to Hawking radiation to AdS/CFT – are still largely governed by the thermodynamics of gravitating systems, not much different from what the simple classical model of N gravitating particles used 40 years ago by Lynden-Bell and Thirring for understanding stellar evolution.

Let me illustrate this by examining several aspects of black hole (BH) physics: I'll divide

these issues into two groups, the first on thermo- statics, such as area law scaling, phase transition and meta-stable states, the second on thermo-dynamics, such as BH formation, BH entropy from collapsing shells and BH nonextensivity.

IV. BLACK HOLE THERMODYNAMICS FROM GRAVITY AS SYSTEMS-WITH-LONG-RANGE-INTERACTION VIEWPOINT

A. Are there substantive differences between a black hole and a self-gravitating gas in their thermodynamics?

After understanding some salient features of gravitational systems from their root up, namely, as systems with negative heat capacity, let us now consider black holes and ask the question: What is the difference between a black hole and a self-gravitating gas in their thermodynamical behavior? One clear distinction is that the red-shift near a black hole event horizon goes to infinity. This gravitational red-shift factor is from the g_{00} component of the metric but one can obtain it from special relativity and avoid general relativity all together, in the same spirit as how Taylor and Wheeler treated the black hole in their book [47]. Thus keep only classical gravity and special relativity (or the equivalence principle, as in [48]), but no GR or quantum physics.

We will start with the same system of self-gravitating gas used by Lynden-Bell but consider the full range of total mass to allow for the formation of a black hole. Let us see if we can obtain Bekenstein's black hole entropy by considering the collapse of a self-gravitating gas. Can we see the BH entropy expression emerging from that of the original gas at the formation of a black hole? This problem was treated by Pretorius, Vollick and Israel [49] who calculated the entropy of a thin spherical shell that contracts reversibly from infinity down to its event horizon and found that, for a broad class of equations of state, the entropy of a non-extremal shell is one-quarter of its area in the black hole limit. (A massive BH was considered to avoid back-reaction effects.) From this the authors gave an operational definition for the entropy of a black hole as the equilibrium thermodynamic entropy that would be stored in the material which gathers to form the black hole, if all of this material were compressed into a thin layer near its gravitational radius. In their derivation they allow for the presence of a quantum field which imparts a Boulware stress-energy in addition to

the classical stress-energy outside the shell. To maintain reversibility, the shell must be in equilibrium with the acceleration radiation seen by observers on the shell. To maintain thermal equilibrium they need to draw on a source of energy at infinity to 'top up' the Boulware stress-energy to an appropriate thermal environment.

This is very nice, yet I would have liked to see a derivation of the black hole entropy-area law without bringing in any quantum field because I believe this law fundamentally describes the macroscopic statistical mechanics property of a classical system with LRI rather than a quantum field-theoretic property which pertains more to the system's microscopic features. This wish of mine seems to be fulfilled in an evocative work of Oppenheim [50] where he showed that the scaling laws of the thermodynamical quantities in the system he studied are identical to those of a black hole, even though the system does not possess an event horizon. His system consists of n densely packed shells supporting itself in its own gravitational field configuration. Generalization to many shells extending the work of Pretorius et al allows the system to be compressed to a size close to its own event horizon. For a system without gravitational interaction the system's entropy is proportional to its volume, no surprise. As weak gravitational interaction is introduced, the system's entropy acquires a correction term, beginning to deviate from the familiar volume-scaling properties. In the limit that the system is about to form a black hole, its entropy is proportional to its area for a large class of equations of state. The scaling laws of the system's temperature and energy are also identical to those of a black hole despite the fact that no horizon is present. The entropy is found to be proportional to the logarithm of the number of micro-states of this system. The temperature, which is usually considered to be independent of the size of the system, is now inversely proportional to the mass of the system. Oppenheim concludes that many of the peculiar properties of black hole thermodynamics are the result of gravitational self-interaction rather than the presence of a horizon.

A related study of interest in purely classical gravity is by [51] on the dynamics of non-spherical gravitational collapse of a bound source allowing for mass loss through gravitational wave emission. The exterior spacetime these authors used is the class of Robinson-Trautmann (RT) metric, which contains the simplest axisymmetric known solutions of Einstein's vacuum field equations representing an isolated gravitational radiating system. For sufficiently smooth initial data, the RT metric converges asymptotically to the Schwarzschild black hole. Physically the bounded source suffers mass loss by gravitational wave emission

until a Schwarzschild black hole is formed. The authors were interested in the relation between the fraction of mass radiated away to the final mass of the bounded source in the nonlinear dynamics described by Einstein's equations. They found that this does not depend on the particular form of the initial data families, but solely on the value of the initial mass, and it satisfies the distribution law for the (Tsallis) nonextensive statistics. This last point brings home the properties associated with LRI systems. Another interesting aspect this work on the critical phenomena of gravitational collapse [52] brings out is the implications for black hole entropy. As we have seen above the black hole entropy is not an extensive quantity, being proportional to its area but not its volume. This is attributed to the long range interaction nature of the gravitational force. If we extrapolate the results of Pretorius et al and Oppenheim to this situation, and use the area of the parameter space for the initial data of the bounded source as a measure of the system's entropy, the area of the formed black hole turns out to be proportional to the black hole entropy. What these authors found is that the final area is always greater than the initial area. They asserted that it might be useful to understand the initial data as an out-of-equilibrium thermodynamical state which evolves towards the equilibrium state identified as the Schwarzschild black hole. They also conjectured that, once a given initial data is specified, the total mass extracted should reflect the nonextensive character of the black hole entropy. This harks back to a striking feature of all systems with LRI [43], the *convergence towards quasi-stationary states*.

I might add two bits of related information here: The mass lost in the form of gravitational waves is measured by the Weyl curvature, the square of which enters in Penrose's definition of gravitational entropy [53] which he introduced to discuss the issue of why the universe started at such a low entropy state. I augmented this classical picture by including quantum matter field in the total entropy budget and posited that particle creation from the dynamics of spacetimes (anisotropic and inhomogeneous, with high gravitational entropy) increases the entropy of matter at the expense of gravitational entropy (smoothing of the universe) [54]. This portion of the gravitational energy and entropy corresponding to gravitational waves (Weyl) is used recently by Chirco and Liberati [55] to fill in some missing content in Jacobson's nonequilibrium thermodynamics formulation of Einstein equation [1], an extension of his earlier equilibrium considerations [2].

In conclusion, let me mention some interesting analog model studies of such systems. It is difficult to perform gravitational experiments in an earth-bound lab on account of

the gravitational system's having negative heat capacity. (One cannot appeal to electric analogs because for an overall charge-neutral system Lebowitz and Lieb [56] showed that for Coulomb systems the microcanonical and canonical ensembles are equivalent.) As we have seen before, a system with negative specific heat is thermodynamically unstable, namely, if it is placed in thermal contact with a second system, even a slight random fluctuation in energy will lower its temperature and increase the energy transfer further. Nevertheless, one can do modeling and simulation with clever setups. Posch and W. Thirring [57] demonstrated these microcanonical features with a simple mechanical model of interacting classical gas particles in a specially confined domain subject to gravitation. The endgame scenario is that most of the gas particles are cooled and collect in the lowest part of the container, where the energy is carried away by a few remaining particles.

Many thermodynamic properties of gravity are due to gravitational interaction being long-ranged. Such properties are also found in small systems. Oppenheim [48] presented a lattice model with long-range spin-spin coupling and showed that the system's temperature and entropy have many properties which are found in black holes. Analog models which can be tested experimentally or simulated theoretically are useful not only to pinpoint the physical origin of the salient features in the thermodynamics of gravitational systems, as is the emphasis of this essay, but once such features are identified they can be used to carry out simulations which are otherwise difficult to perform for gravitational systems.

V.   DISCUSSIONS

Two most familiar models of emergent theories for an average physicist like me are probably hydrodynamics and thermodynamics which describe the robust or stable macroscopic manifestations of a microscopic theory (or many). Hydrodynamics is robust because of the existence of conservation laws for the collective or hydrodynamic variables. Thermodynamics describes stationary configurations under equilibrium conditions determined by the maximal entropy principle. In my search for understanding the emergent behavior traversing the familiar macroscopic world into the unknown microscopic structures of spacetime (quantum gravity) I have relied more, as a conceptual scheme, on hydrodynamics than thermodynamics, both are macroscopic manifestations of the micro-theory of molecular dynamics, thus my thoughts on the kinetic theory approach to quantum gravity [24] and spacetime as con-

densate [25] concepts. I have postponed thinking about the thermodynamics of spacetime because while I find it to be more powerful it is in other ways more restrictive and thus less representative: powerful in what are contained in the laws of thermodynamics, restrictive in the assumption of the equilibrium condition. I feel that if we are to describe the dynamics of spacetime via Einstein equations [1] or Newton's equation of motion of a particle [3] through thermo- or hydro-dynamics we need to consider nonequilibrium (NEq) conditions ab initio. This explains why I place more emphasis on the nonequilibrium dynamics and thermodynamics for gravitational systems. They are in fact intrinsically nonequilibrium. Yet of course there is more work devoted to the thermodynamics because we know a great deal more about the equilibrium conditions for the dynamics of both matter and spacetime than the nonequilibrium conditions. Before we end this essay it may therefore be appropriate to discuss what nonequilbrium means for the problems raised here versus the problem tackled in the thermo-gravity theories of JPV [1–4]. For matter it is straightforward as can be found in any textbook on nonequilibrium statistical mechanics. For quantum vacuum it is more subtle and tricky – see d) below.

a) Gravity is intrinsically non-equilibrium: the reader probably first encounters this feature from the Jeans instability. Black hole (equilibrium) thermodynamics is inadequate for treating important issues such as the end state and information loss issues. Many salient features of gravity stem from the fact that it has negative heat capacity which implies that no stable equilibrium configuration can exist in an isolated gravitational system. Rather, it requires dynamical considerations (evolution with backreaction) and nonequilibrium descriptions. As I advocated at the start, it is more natural and productive if we treat and view gravitational phenomena in the idioms and tenets of nonequilibrium physics. What this entails is, understand the general features of NEq physics more thoroughly, then analyze gravitational systems in that light. We should try to understand better all thermodynamical properties of gravitational systems from this perspective.

b) Rely only on the basics: Instead of tapping into exoteric themes to derive the gravitational equations of motion, we should perhaps first try to invoke only, but delve deeper into, the nonequilibrium statistical mechanics of gravitating systems.

Indeed let me pose this as a challenge to the young audience here: try to reproduce as many known facts about the thermodynamics of gravitating systems, in particular, black hole thermodynamics, from the NEq thermodynamics of systems with long range interactions

(LRI). The part which remains after this filtration is what truly distinguishes black hole thermodynamics from that of other LRI systems.

c) Differing philosophies: In observance of the Austerity Principle and the Commonality Principle we want to find out what in the familiar black hole thermodynamics cannot be obtained from the more basic and generic features of LRI which places gravity under the same roof with many other physical systems. This way of thinking has several advantages: 1) Philosophically it is gratifying to see all the implications stemming from one source rather than requiring many ingredients all taken into account with the same degree of importance. What does not fall under these generalities can then be identified as unique about gravity. 2) To the extent that gravity behaves similar to, say, certain condensed matter systems, the theories describing these more familiar systems can serve as models to discover new phenomena. 3) Experiments related to the analog systems with similar properties can be used to test out predictions in gravitating systems which are difficult to check experimentally. (For example, laboratory cosmology [58].)

d) Gravity related to the nonequilibrium matter vs gravity related to the quantum field vacuum: Two distinct levels of inquiry

What we have studied in this paper is classical matter for which the ordinary concepts and techniques of nonequilibrium statistical mechanics apply. This is very different from the subject of investigation in the recent proposal [3] and the original theory [1] where the nature of gravity in the quantum field vacuum is studied. Concept of equilibrium in that context relies on the Rindler vacuum being thermal with respect to a Minkowski observer, thus invoking the results of Fulling, Davies and Unruh. In [1] the equilibrium considered is the local vacuum, viewed from the neighborhood of the bifurcation plane of a local Rindler wedge. No global symmetry or uniform acceleration is invoked there in contrast to [3], except in the small neighborhood of each point of spacetime. Nonequilibrium refers to the notion of local causal horizons with shear, in which case the system is further from equilibrium than for local horizons without shear. Note also the 'system' under consideration in [1] is defined not as all of the gravitational field, but by the partition created by the local horizon. How does one connect these concepts of temperature and equilibrium associated with a Rindler horizon with the familiar concepts of nonequilibrium statistical mechanics of matter is perhaps the first issue which needs clarification before one can talk freely about the thermodynamics of spacetime and from there derive equations of motion for matter.

Appendix: Critique on Gravity as Entropic Force [3]

Force in polymer chain, crumpling transition. $F\Delta x = T\Delta S$. Ingredients:

1) Bekenstein-Hawking entropy, *holography principle* (screen, etc): Particle at distance of a Compton wavelength $\Delta x = h/mc$ from the BH horizon would increase the entropy of the BH by one bit $\Delta S = 2\pi k$ (Bekenstein considered a particle fallen into the BH)

2) Relating temperature T to acceleration a via Unruh effect $T = \hbar a/ck$, get *Newton's Second Law* $F = T(\Delta S/\Delta x) = ma$.

3) *For Newton's Law of Gravitation:* Invoke holography: max number of bits storage space is proportional to the area of screen: $N = A(c^3/G\hbar)$. Relating energy E or mass M of star (enclosed by screen) to N and T by invoking the equipartition theorem $E = 1/2NkT = Mc^2$: counting the number of degrees of freedom on the event horizon [4]. Replace T by E/N and get $F = T(\Delta S/\Delta x) = GMm/(2\pi)R^2$. $(Area A = 4\pi R^2)$

My Critique – I'll mention just three for now, one pertaining to the overall intent, two pertaining to central principles:

1) *Why do we need quantum physics from the Compton wavelength of a particle to big ideas like holography or quantum information theory to get mere classical gravity?* This can be brushed off as a matter of preference depending on what ingredients one views as more basic and thus more important. The proponent's argument is that these features come with quantum field theory and if one can find the root of gravity in these more primitive concepts it is a gain. Preferences can only be persuaded but not be debated. Yet this question gains gravity and becomes more demanding after what we have seen explained above, i.e, many salient features of gravity are shared by systems with NHC or LRI. Take glassy system for example, do we have to go through this labyrinth of intricate ideas and constructions involving entropy-area, holography or (the equivalent of) Unruh temperature applied to local Rindler observer's vacuum to obtain the equations of motion for the dynamics of ordinary glass? What does one gain in this conceptual maze over the conventional ways in condensed matter physics?

2) *Holography and quantum information*: The area law: entropy $S \sim A$ is a rather generic feature and has been derived in many ways for various physical systems [59–61], not just for black holes. In the most commonly relatable way it can be viewed as a consequence of

partitioning a closed system into subsystems and counting the resultant subsystem's degrees of freedom (e.g., [62]). The projection operator formalism /concepts is quite basic in NEq statistical mechanics. One does not need to invoke big ideas like the holography principle nor new tenets from quantum information to get or appreciate these results.

3) *Thermality of vacuum and dynamics:* Invoking the thermal equilibrium condition for the description of dynamics is, at least by common sense, a bit of a stretch. Arbitrary motion has no thermodynamic description, temperature is ill-defined. Concept of temperature is meaningful only under very special conditions, e.g., Unruh effect manifests only for a detector in uniform acceleration. General state of motion is under nonequilibrium conditions [28] and its full content can only reveal by using nonequilibrium quantum field theory [63]. As explained above the notion of equilibrium in the thermo-gravity proposals refers to the local Rindler vacuum which is very different from the notion of equilibrium referring to classical matter or even quantum fields.

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[64] 'Chop-Sui' is a nondescript Chinese dish invented a century ago in the American Chinatown ghettos which can neither be found nor even be heard of anywhere in China. Literally it means "cut the miscellanies and mix them up".